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ABSTRACT

This is the second of two guidebooks on minimum course content for high school geometry, and is designed for the student who has mastered the skills and concepts of Geometry I and who had a final average of low B or less. Emphasis is on understanding and use of theorems without proof. This course develops definitions and properties of triangles, quadrilaterals, circles, polygons, and solid figures. Methods for finding linear measures, lateral and total measures, and volume measures are formulated; the Pythagorean Theorem and special right triangle relationships are developed. Overall course goals are stated, then for each of the topics there is a list of performance objectives, textbook references, course content, and suggested learning activities. Sample posttest items, an annotated bibliography of 16 books, and a list of films, filmstrips, and transparencies are included. (DT)

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# AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

GEOMETRY 2

5218.22

Mathematics

DIVISION OF INSTRUCTION•1971

SE 016 506

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

GEOMETRY 2

5218. 22

(EXPERIMENTAL)

Written by

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for the

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Dade County Public Schools
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1971-72

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# PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for the class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

## CATALOGUE DESCRIPTION

The second of the two-quin sequence which introduces the student to all of the theorems usually included in high school geometry. Emphasis is on understanding and use of these theorems without proof. Develops definitions and properties of plane and solid figures. Formulates methods for finding their linear measures, lateral and total measures, and volume measures. Develops the Pythagorean Theorem and special right triangle relationships.

Designed for the student who has mastered the skills and concepts of Geometry 1, with a final average of low B or less.

## TABLE OF CONTENTS

## OVERALL GOALS

The student will:

1. Review linear measure and extend his measurement knowledge to include area and volume.

2. Improve his ability to visualize and sketch two and three-dimensional figures.

3. Build an intuitive understanding of the properties of plane and solid figures, including triangles, quadrilaterals, other polygons, circles, prisms, cylinders, cones, spheres, and compound figures.

4. Develop and apply the many formulas related to the forementioned plane and solid figures.

5. Correlate the appropriate scientific principles with their mathematical applications in this quin. (Cavalierri's principle and Archimedes' Principle).

6. Enrich his appreciation of modern geometric construction through the comparative study of methods used throughout history.

7. Strengthen his preparation for future science courses through stress on the measurement formulas introduced in this quin; in particular, the special right triangle and volume formulas.

8. Improve his ability to reason informally and add to his foundation for future development of formal proof.

9. Extend his use of mathematical symbols, notations, and vocabulary necessary to the material introduced in this quin.

10. Increase his speed and accuracy in computation.

11. Add to his special reading techniques for mathematics and science.

## KEY TO STATE ADOPTED REFERENCES


M - Moise, Edwin E. and Downs, Floyd L., Jr. Geometry. Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1967.

L - Lewis, Harry. Geometry, A Contemporary Course. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1968.

[illegible] - Jurgensen, Donnelly, Doliciani. Modern School Mathematics Geometry. Boston: Houghton Mifflin Company, 1966.

A - Anderson, Garon, Gremillion. School Mathematics Geometry. Boston: Houghton Mifflin Company, 1966.

I. TRIANGLES

A. KINDS AND RELATED PARTS

## PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items selected from the vocabulary list, labeling them correctly on appropriate diagrams.

2. Develop and apply items selected from the list of associated properties given under Course Content.

3. Compute the measure of the third angle of a triangle, given the measures of two angles.

4. Find the measure of an exterior angle of a triangle, given appropriate information.

5. Find the measure of a remote interior angle of a triangle, given appropriate information.

6. Construct a triangle with a straight edge and compass given:

   a. SSS
   b. ASA
   c. SAS
   d. SAA
   e. HL of rt $\Delta$
   f. HA of rt $\Delta$

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTER | 4, 5, 7, 9 | 5, 10, 15 | 5, 6 | 5 |

## COURSE CONTENT

Vocabulary

polygon
triangle
  side
  angle
  vertex
  adjacent sides
  scalene
  isosceles
equilateral
equiangular
exterior angle
included angles
included sides
exterior
interior
right
obtuse
acute
legs
hypotenuse
vertex angle
remote interior angle
base
base angles

I. TRIANGLES

A. KINDS AND RELATED PARTS

Associated Properties

A triangle is a three-sided polygon.

If two sides of a triangle are congruent, the angles opposite those two sides are congruent; and, conversely, if two angles of a triangle are congruent, the sides opposite those angles are congruent.

Every equilateral triangle is equiangular and vice versa.

For every triangle, the sum of the measures of the angles is $180^{\circ}$.

The acute angles of a right triangle are complimentary.

If two pairs of angles of two triangles are contruent, then the third pair of angles is congruent.

For any triangle, the measure of an exterior angle is the sum of the measures of its two remote interior angles.

Constructions

Construct a triangle with a straight edge and compass given:

a. SSS
b. ASA
c. SAS
d. SAA
e. HL of rt $\Delta$
f. HA of rt $\Delta$

SUGGESTED LEARNING ACTIVITIES

1. Cut out and fold paper models of triangles to demonstrate measure relationships of the angles of a triangle.

2. Repeat the above constructions using protractor and ruler and compare to the constructions done with compass and straightedge.

I. TRIANGLES

B. CONGRUENT TRIANGLES

## PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items selected from the vocabulary list.
2. Label corresponding parts of given marked figures.
3. Name congruent triangles using notation which shows corresponding vertices of a given marked figure.
4. State the postulate which justifies the congruence of given pairs of triangles.
5. Identify all corresponding parts of given congruent triangles, where the correspondence is shown:
   a. pictorially
   b. by letters without a figure
6. With only a compass and straightedge copy a given triangle using each of the following methods:

| | |
|---|---|
| a. SSS | d. SAA |
| b. SAS | e. HL of a rt $\triangle$ |
| c. ASA | f. HA of a rt $\triangle$ |

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTER | 4, 5 | 5 | 6 | 6, 7 |

## COURSE CONTENT

Vocabulary

| | |
|---|---|
| one-to-one correspondence | rotate |
| corresponding sides | flip |
| corresponding angles | slide |
| congruent sides | rigid |

I. TRIANGLES

B. CONGRUENT TRIANGLES

Vocabulary (continued)

congruent angles

congruent triangles

identity congruence

self congruence

Associated Properties

Two polygons are congruent if, for some pairing of their verticles each side and each angle of one polygon is congruent to the corresponding part of the other polygon.

The correspondence ABC$\leftrightarrow$ABC is called the identity congruence.

If AB $\cong$ AC, then the correspondence ABC$\leftrightarrow$ABC is called a self congruence, which is not an identity congruence.

Congruence postulates and theorems: SSS, ASA, SAS, SAA, HL, HA

Constructions

Copy a triangle if given: SSS, ASA, SAS, SAA, HL of a rt $\triangle$, HA of a rt $\triangle$

SUGGESTED LEARNING ACTIVITIES

1. Cut and fold paper models to demonstrate correct correspondence in congruent figures. (Especially helpful in illustrating self-congruence in polygons with two or more congruent sides.)

2. Use filmstrip A542-4 by S.V.E.

3. Use filmstrips 1122 and 1157 by FOM

4. Use overhead transparency master #9 to accompany Moise-Downs Geometry

5. Use selected overhead visuals for Geometry by Jurgensen, Donnelly, Dolciani, Vol. III & V

I. TRIANGLES
C. INEQUALITIES

## PERFORMANCE OBJECTIVES

The student will:

1. Use the order properties (transitivity, addition, multiplication by a positive number) in the solution of simple inequalities.

2. Find the missing angle measures of a triangle if given measures of an exterior angle and one remote interior angle.

3. Use exterior angle properties of a triangle and transitivity to establish relative sizes of angles in a given figure.

4. Determine the largest angle of a triangle if given measures of the sides.

5. Determine the longest side of a triangle if given the measures of the angles.

6. Determine the largest angle of a figure partitioned into triangles if given appropriate information.

7. Determine the longest side of a figure partitioned into triangles if given appropriate information.

8. Solve numerical problems involving the definition of distance from a point to a line or plane.

9. Determine whether three given measures could be the lengths of the sides of a triangle.

10. Use the "Hinge Theorem" and its converse to compare lengths of segments and measures of angles in 2 triangles.

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTER | 7 | 16 | 7 | 8 |

I. TRIANGLES
C. INEQUALITIES

## COURSE CONTENT

### Vocabulary

order of inequality

### Associated Properties

Properties of order:

| | |
|---|---|
| trichotomy | addition |
| transitivity | multiplication by a positive number |

An exterior angle of a triangle is larger than each of its remote interior angles.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

If a triangle has one right angle, then its other angles are acute.

If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent and the larger angle is opposite the longer side; and, conversely.

The shortest segment joining a point and a line is the perpendicular segment from the point to the line.

Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Hinge Theorem: If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is larger than the indluded angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle; and, conversely.

The distance from a point to a line or from a point to a plane is the length of the perpendicular segment.

I. TRIANGLES
C. INEQUALITIES

## SUGGESTED LEARNING ACTIVITIES

1. Use 2 blackboard compasses with different angle measures against the blackboard to demonstrate the hinge theorem and its converse. If desired, the third side may actually be drawn on the blackboard.

2. Draw a large triangle with all sides extended. Number all angles. Use a pointer to indicate an angle which the class must decide is or is not an exterior angle of the triangle. If they decide it is, they must call out the numbers of its remote interior angles.

3. Have selected students construct models to demonstrate:

   a. the shortest segment joining a point and a line
   b. the Triangle Inequality Theorem
   c. the Hinge Theorem and its converse

I. TRIANGLES
D. PERIMETER AND AREA

## PERFORMANCE OBJECTIVES

The student will:

1. Find the perimeter of a triangle given the measures of the three sides.

2. Find the length of a side of a triangle given the perimeter and the measures of the two sides.

3. Find the smallest number of triangular regions into which a given polygonal region can be divided.

4. Find the area of a triangle given the necessary measures.

5. Find a missing part of a triangle given the area and necessary measures.

6. Express the area of a triangle in terms of its perimeter and measures of its sides. (Hero's Formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$
$s = 1/2\,(a+b+c)$)

7. Compare the areas of two triangles having a) equal bases and altitudes b) equal bases c) equal altitudes.

## STATE ADOPTED REFERENCES

| | M | T | JD | A |
|---|---|---|---|---|
| CHAPTERS | 9 (254-256)<br>2 (23-24)<br>12 (352, Ex.14) | 11 | 9 | 13 |

## COURSE CONTENT

Vocabulary

| | | |
|---|---|---|
| triangular region | perimeter | triangulate |
| polygonal region | area | |
| altitude | ratio | |

I. TRIANGLES

D. PERIMETER AND AREA

Associated Properties

If two triangles are congruent, then the triangular regions determined by them have the same area.

To every polygonal region there corresponds a unique positive number.

Given that the region R is the union of two regions $R_1$ and $R_2$, and $R_1$ and $R_2$ intersect in at most a finite number of points and segments, then $aR = aR_1 + aR_2$.

The area of a square region is the square of the length of its edge. (Emphasize as unit of measure).

The area of a rectangle is the product of the lengths of its base and its altitude.

The area of a right triangle is half the product of the length of its legs.

The area of a triangle is half the product of the lengths of any base and the corresponding altitude.

The perimeter of a triangle is equal to the sum of the measures of its sides.

The area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where $s=1/2\,(a+b+c)$

If two triangles have the same base and equal altitudes, then they have the same area.

If two triangles have the same altitude, then the ratio of their areas is equal to the ratio of their bases.

SUGGESTED LEARNING ACTIVITIES

1. Have students bring in scale drawings of their rooms and compute the cost of: a. wallpaper or painting
   b. carpeting

2. Have students make scale drawings of playing regions for popular sports (tennis, football, baseball, etc.). Devise an appropriate list of questions based on these drawings.

I. TRIANGLES

D. PERIMETER AND AREA

3. Make a model for demonstrating triangular area using a rigid base segment and a track parallel to the base segment a convenient distance above it. Use a nail in track and elastic attached to endpoints of base. Slide nail along track to demonstrate triangles with different shapes but same area.

4. Overhead transparency #17 - Moise-Downs: Geometry.

I. TRIANGLES

E. PYTHAGOREAN THEOREM AND SPECIAL RIGHT TRIANGLE RELATIONSHIPS AND APPLICATIONS

## PERFORMANCE OBJECTIVES

The student will:

1. Apply the Pythagorean Theorem to find the missing measure of the sides of a given right triangle.

2. Strengthen his ability to simplify radicals.

3. Use the converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

4. Apply the appropriate formulas to solve for the missing parts in special right triangles. (30-60-90 and 45-45-90)

5. Demonstrate a minimum of three developments of the Pythagorean Theorem.

6. Develop and apply the formula for the area of an equilateral triangle

7. Determine the lengths of the sides and the altitude of an equilateral triangle given the area of the triangle.

8. Memorize the three most used Pythagorean triplets: 3, 4, 5; 5, 12, 13; and 8, 15, 17.

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTERS | 9 (254-256)<br>11<br>2 (pp. 23-24)<br>12 (pg. 352 ex. 14) | 11 | 9 | 13 |

I. TRIANGLES

E. PYTHAGOREAN THEOREM AND SPECIAL RIGHT TRIANGLE RELATIONSHIPS AND APPLICATIONS

Vocabulary

| | |
|---|---|
| square | radicand |
| square root | right angle |
| radical | isosceles right triangle |
| index | |

Associated Properties

Pythagorean Theorem and its converse.

Special right triangle relationships and converses. (30-60-90) (45-45-90)

The altitude to the hypotenuse of a right triangle in terms of its legs is equal to: $\frac{l_1 l_2}{\sqrt{l_1^2 + l_2^2}}$.

The area of an equilateral triangle equals $\frac{S^2}{4}\sqrt{3}$

SUGGESTED LEARNING ACTIVITIES

1. Have students construct various right triangles with squares upon the three sides. Have them measure and compute areas of the squares to develop the Pythagorean relationship.

2. Have students present reports on historical background of the Pythagorean Theorem.

3. Use filmstrip by FOM #1168 Special Right Triangles

4. Use overhead transparency #19, Moise-Downs, Geometry.

I. TRIANGLES

F. SIMILAR TRIANGLES

## PERFORMANCE OBJECTIVES

The student will:

1. Find the missing term of a proportion when given the other three terms.

2. Find the geometric and arithmetic mean of two terms given in numerical or literal form.

3. Determine whether figures in a given marked diagram are similar; and if so, identify the property which justifies the conclusion.

4. List all proportions determined by a line intersecting two sides of a triangle and parallel to the third side.

5. Determine whether a segment is parallel to one side of a triangle if given certain proportional parts.

6. Identify the proportions determined by a bisector of an angle of a triangle.

7. Solve for the missing measures of a right triangle if given the measure of the altitude to the hypotenuse and one other measure.

8. If given the ratio of corresponding parts of similar figures and other appropriate information, apply the proper formulas to find:

   a. perimeter
   b. area
   c. volume

9. Apply the definition of similarity of triangles to write a proportion and solve for an unknown length of a side, given the appropriate information.

10. Use similar triangles to solve problems involving indirect measurement.

I. TRIANGLES

F. SIMILAR TRIANGLES

STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTERS | 11, 12 | 11 | 8, 9 | 13, 14 |

COURSE CONTENT

Vocabulary

means
extremes
geometric mean
arithmetic mean
projection
similar polygons
similar triangles
proportional segments

Associated Properties

Two polygons are similar if, for some paring of their vertices, corresponding angles are congruent and corresponding sides are in proportion.

Similarity theorems: AAA, AA, SAS, SSS

If a line is parallel to one side of a triangle and intersects the other sides, then a) it divides the sides into proportional segments; b) it cuts off segments proportional to the given sides.

Transitivity of similarity with respect to triangles.

The altitude to the hypotenuse of a right triangle forms two triangles which are similar to the given triangle and similar to each other.

The altitude to the hypotenuse of a right triangle is the mean proportional (geometric mean) between the segments into which it divides the hypotenuse.

A leg of a right triangle is the mean proportional between the hypontenuse and the projection of that leg on the hypotenuse.

If two triangles are similar, then the ratio of their areas is equal to the square of the ratio of any two corresponding sides, altitudes, or medians.

I. TRIANGLES

F. SIMILAR TRIANGLES

Associated Properties (continued)

If two triangles are similar, then the ratio of their perimeters is equal to the ratio of any two corresponding sides, altitudes, or medians.

## SUGGESTED LEARNING ACTIVITIES

1. Use Film Catalogue #1-01506 Similar Triangles

2. FOM Filmstrips
   1161 Mean Proportion and Right Angles
   1116 Similar Triangles-Experiment and Deduction

3. Overhead transparencies Moise-Downs #21, 22

4. Have students bring in examples of similarity relationships such as: model cars, model planes, enlarged photograph, house plans, doll clothes and furniture, or any blueprints, maps, etc.

5. Cut out paper models of figures and slide them over each other to demonstrate similarity and the constant ratio of corresponding sides.

I. TRIANGLES

G. CONSTRUCTIONS

## PERFORMANCE OBJECTIVES

The student will:

1. Using only a compass and straightedge, construct:

   a. Isosceles triangles
   b. Equilateral triangles
   c. Right triangles
   d. Angle bisectors of triangles
   e. Perpendicular bisectors of the sides of a triangle
   f. Medians of a triangle
   g. Altitudes of a triangle
   h. Inscribed circle of a triangle
   i. Circumscribed circle of a triangle

2. Demonstrate the difference in the terms "sketch" and "construct".

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTERS | 15 | 11, 15 | 6, 11 | 13, 17 |

## COURSE CONTENT

### Vocabulary

Set of points related to a triangle:

angle bisector
perpendicular bisector of a side
median
circumcenter
incenter
orthocenter
centroid
circumscribed circle
inscribed circle

### Associated Properties

The medians of every triangle are concurrent at a point two-thirds of the distance from one vertex to the mid-point of the opposite side.

The segment joining the midpoints of two sides of a triangle is parallel to the third side and one-half as long.

I. TRIANGLES
G. CONSTRUCTIONS

Associated Properties (continued)

The median to the hypotenuse of a right triangle is half as long as the hypotenuse.

In a triangle, if a median is half as long as the side which it bisects, then the triangle is a right triangle and the side is its hypotenuse.

## SUGGESTED LEARNING ACTIVITIES

1. Have sutdents report on development of construction implements from collapsible compass to modern day drafting tools.

2. Have students report on trisection on an angle.

3. Have students do constructions by two methods:

   compass and straightedge
   ruler and protractor

II. QUADRILATERALS

A. KINDS AND RELATED PARTS

## PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items from vocabulary list.

2. Identify and sketch items from vocabulary list, if given their definitions.

3. Label and identify parts of figures correctly.

4. Draw a Venn diagram to illustrate the relationships of quadrilaterals and their subsets.

5. Develop and apply the associated properties listed under "Course Content" to numerical and non-numerical exercises.

6. Tell if given information is sufficient to classify a quadrilateral as a trapezoid or an isosceles trapezoid.

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTERS | 5, 9 | 8, 17 | 5, 7, 15 | 10, 13 |

## COURSE CONTENT

Vocabulary

quadrilateral
opposite angles
consecutive or successive angles
opposite sides
adjacent sides
consecutive or successive sides
diagonal
convex quadrilateral
vertices
clockwise
counterclockwise
supplementary angles
parallel sides
rectangle
square
trapezium
trapezoid
rhombus
base angles of trapezoid
kite
trisect
isosceles trapezoid
median of a trapezoid

II. QUADRILATERALS

A. KINDS AND RELATED PARTS

Associated Properties

Every quadrilateral is a four-sided polygon.

The sum of the measures of the angles of a quadrilateral is $360^{\circ}$

The median of a trapezoid is parallel to the bases and equal in measure to one-half the sum of the bases.

The median of a trapezoid bisects both diagonals.

If two consecutive angles of a trapezoid are congruent but not supplementary, the trapezoid is isosceles.

The bisectors of the opposite angles of a nonrhombic parallelogram are parallel.

(Optional) If a trapezoid has two nonparallel sides both congruent to one of the parallel sides, then the diagonals bisect the angles at the other parallel side.

SUGGESTED LEARNING ACTIVITIES

1. Chart of quadrilaterals and their properties may be begun here and continued throughout II - B.

2. Cut out, fold and measure paper models etc.

3. Use Venn Diagrams to help student learn properties of quadrilaterals.

4. Have students construct models using hinged sides and elastic to demonstrate properties of special quadrilaterals.

II. QUADRILATERALS

B. PARALLELOGRAMS

## PERFORMANCE OBJECTIVES

The student will:

1. Define and illustrate items from vocabulary list.

2. State properties of all parallelograms.

3. State special properties of rectangle, rhombus, and square.

4. Given a property, name all quadrilaterals which have that property.

5. Complete a chart of all quadrilaterals and their properties.

6. Tell if given information is sufficient to classify a quadrilateral as a parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid and/or kite.

7. State all sets of conditions which would guarantee that a quadrilateral is a parallelogram, rectangle, rhombus, square, trapezoid, isosceles trapezoid and/or kite.

8. Determine the measures of three angles of a parallelogram, given the measure of one angle

9. Determine the missing measures of angles in compound plane figures composed of triangles and quadrilaterals.

10. Determine the missing lengths of sides and/or diagonals given necessary and sufficient information.

11. Name the figure formed by joining the consecutive mid-points of the sides of:

    a. any quadrilateral
    b. a parallelogram
    c. a rectangle
    d. a rhombus
    e. a square

12. Determine if sufficient, insufficient or too much information is given to find the solution of a problem.

II. QUADRILATERALS
B. PARALLELOGRAMS

## COURSE CONTENT

### Associated Properties

Each diagonal of a parallelogram separates a parallelogram into two congruent triangles.

Any two opposite sides of a parallelogram are congruent.

Any two opposite angles of a parallelogram are congruent.

Any two consecutive angles of a parallelogram are supplementary.

The diagonals of a parallelogram bisect each other.

If the bisectors of two consecutive angles of a parallelogram intersect, they are perpendicular.

Given a quadrilateral in which both pairs of opposite sides are congruent. Then the quadrilateral is a parallelogram.

If two sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

The segments joining the mid-points of opposite sides of any quadrilateral bisect each other.

If a parallelogram has one right angle, then it has four right angles, and the parallelogram is a rectangle.

In a rhombus, the diagonals are perpendicular to one another.

If the diagonals of a quadrilateral bisect each other and are perpendicular, then the quadrilateral is a rhombus.

The diagonals of a rectangle are congruent.

The diagonals of a rhombus bisect the angles of the rhombus.

II. QUADRILATERALS

B. PARALLELOGRAMS

Associated Properties (continued)

If three parallel lines intercept congruent segments on one transversal T then the segments they intercept on every transversal $T_i$ which is parallel to T, are congruent to each other and to the segments of T.

If three or more parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any other transversal.

A kite has two pairs of congruent sides, but its opposite sides are not congruent.

The figure formed by the segments joining the mid-points of consecutive sides of any:

(a) quadrilateral is a quadrilateral
(b) parallelogram is a parallelogram
(c) rectangle is a rhombus
(d) rhombus is a rectangle
(e) square is a square

If two lines are parallel, then all points of each line are equidistant from the other line.

Every parallelogram is an isosceles trapezoid.

## SUGGESTED LEARNING ACTIVITIES

1. Continue activities #1-4, from II A

2. Filmstrip by FOM #1113 (Parallelograms and Their Properties)

II. QUADRILATERALS

C. PERIMETER AND AREA

## PERFORMANCE OBJECTIVES

The student will:

1. Develop and apply the standard formulas for finding the perimeter of any quadrilateral, any parallelogram, special parallelograms, trapezoids, kites and compound figures.

2. Develop and apply the standard formulas for finding the area of any quadrilateral, any parallelogram, special parallelograms, trapezoids, kites and compound figures.

3. Find missing lengths of parts necessary to the use of standard perimeter and area formulas, given appropriate angle measures and lengths of segments.

4. Find measures of diagonals, bases and/or altitudes, given other appropriate measures.

5. Find ratio of perimeters, areas, sides, and/or altitudes of figures.

6. Find the perimeter of a figure, given the perimeter of another figure and the ratio of the two perimeters or other parts.

7. Find the area of a figure, given the area of another figure and the ratio of the two areas.

8. Apply metric units of measure to the above objectives where appropriate.

## COURSE CONTENT

Vocabulary

base
parallel
square unit
linear unit
compound figures

II. QUADRILATERALS
    C. PERIMETER AND AREA

## Associated Properties

The perimeter of a square, rectangle, rhombus, parallelogram, trapezoid or any quadrilateral is equal to the sum of the lengths of its sides.

The area of a square equals the square of the length of a side.

The area of a rectangle equals its base times its height (altitude).

The area of a parallelogram equals its base times its altitude.

The area of a trapezoid equals $1/2h\ (b_1+b_2)$.
$h$ = height
$b_1$ and $b_2$ = lengths of bases

If the diagonals of a rhombus, square of kite are $d_1$ and $d_2$, then the area of the enclosed region is $1/2 d_1\ d_2$.

If the diagonals of a convex quadrilateral are perpendicular to each other, then the area of the quadrilateral equals one-half the product of the lengths of the diagonals.

## SUGGESTED LEARNING ACTIVITIES

1. Use cut out figures to illustrate area formulas and relate the area of a parallelogram to that of a rectangle by replacing the right triangle, etc.
2. Overhead transparency Moise-Downs Geometry #18
3. Distribute cardboard cutouts of various quadrilaterals and have students measure required parts and compute perimeter and area.
4. Discussion Question: Does perimeter determine area? Take 4 strips of cardboard 1/2 inch by 4 inches and fasten them together so that they make a square. Adjust the frame to different positions and trace the inside outline on graph paper. Estimate the area of each figure.

II. QUADRILATERALS
D. CONSTRUCTIONS

## PERFORMANCE OBJECTIVES

Usingonly a compass and straightedge, the student will construct:

1. A parallelogram starting with (a) its diagonals (b) a pair of consecutive sides (c) a pair of parallel segments (d) a side, one acute angle, and the longer diagonal given (e) 2 segments and 1 angle given, representing 2 sides and the included angle.

2. A rhombus given (a) two line segments representing its diagonals, (b) a segment and an angle representing its sides and one angle.

3. A square

4. A rectangle

5. A trapezoid

6. A kite

7. An altitude from any given vertex of a quadrilateral

8. A median of a trapezoid

9. A rhombus using a given line segment of lengths as one side and a given angle A as one of the angles.

## COURSE CONTENT

### Associated Properties

Four sets of conditions which determine a parallelogram are:

1. The diagonals of a quadrilateral bisect each other.
2. Both pairs of opposite sides of a quadrilateral are parallel.
3. Both pairs of opposite sides of a quadrilateral are congruent.
4. One pair of opposite sides of a quadrilateral are congruent and parallel.

II. QUADRILATERALS
D. CONSTRUCTIONS

## SUGGESTED LEARNING ACTIVITIES

1. Repeat selected constructions using modern construction implements:
   a) T square
   b) ruler
   c) protractor

2. As a review or a test, distribute envelopes containing construction instructions for different quadrilaterals to each student, and have him construct the required quadrilateral using straightedge and compass.

3. Throughout this quin, the teacher should include more challenging construction problems from the State-adopted texts and from the sources listed in the annotated bibliography at the end of this quin.

III. CIRCLES

A. RELATED PARTS

## PERFORMANCE OBJECTIVES

The student will:

1. Identify and label the related parts of a circle in a given diagram.

2. Draw a diagram of a circle and its related parts and label the figure, given a list of the required parts.

3. Name the related parts of a circle, given their definitions.

4. Define items from the vocabulary list.

5. Solve numerical and non-numerical problems based on items in the vocabulary list and the associated properties list.

6. Develop items selected from the list of associated properties intuitively.

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTERS | 14, 15, 16 | 14, 15, 17 | 10, 15 | 16, 17, 18 |

## COURSE CONTENT

Vocabulary

circle
center
sphere
radius
compass
straightedge
diameter
chord
tangent (line, segment, ray)
secant (line, segment, ray)
point of tangency
circumference
concentric circles
great circle
congruent circles
segment
sector
semi-circle
arc
minor arc
major arc
intercepted arc
inscribed polygon
circumscribed polygon

III. CIRCLES

A. RELATED PARTS

Vocabulary (continued)

internally tangent
externally tangent
common internal tangent
common external tangent
pi ($\pi$)
irrational
arc in which angle is inscribed

Associated Properties

Pi is the constant ratio of the circumference of a circle to its diameter.

The diameter equals twice the radius.

The intersection of a sphere with a plane through its center is a circle with the same center and the same radious as the sphere.

A line perpendicular to a radius at its outer end is tangent to the circle.

Every tangent to a circle is perpendicular to the radius drawn to the point of contact.

The perpendicular from the center of a circle to a chord bisects the chord.

The segment from the center of a circle to the mid-point of a chord is perpendicular to the chord.

In the plane of a circle, the perpendicular bisector of a chord passes through the center.

If two circles are tangent, their centers are collinear with their point of tangency.

No circle contains three collinear points.

In the same circle or in congruent circles, chords equidistant from the center are congruent.

If a line intersects the interior of a circle, then it intersects the circle in two and only two points.

III. CIRCLES

A. RELATED PARTS

Associated Properties (continued)

In any circle, the mid-points of all chords congruent to a given chord form a circle concentric with the given circle and with a radius equal to the distance of any one of the chords from the center.

In a circle, if two chords which have a common end point determine congruent angles with a diameter from the same point, then the chords are congruent.

If two congruent chords of a circle intersect on a diameter, they determine congruent angles with the diameter.

Any three noncollinear points lie on a circle.

A plane perpendicular to a radius at its outer end is tangent to the sphere.

Every tangent plane to a sphere is perpendicular to the radius drawn to the point of contact.

If a plane intersects the interior of a sphere, then the intersection of the plane and the sphere is a circle. The center of this of this circle is the foot of the perpendicular from the center of the sphere to the plane.

The perpendicular from the center of a sphere to a chord bisects the chord.

The segment from the center of a sphere to the mid-point of a chord is perpendicular to the chord.

If two diameters of a sphere are perpendicular, the figure formed by the segments joining their end points in succession is a square.

Any two great circles of a sphere intersect at the end points of a diameter of the sphere.

If two planes intersect a sphere and their distances from the center are equal, then the intersections are either two points or two congruent circles.

III. CIRCLES

A. RELATED PARTS

Associated Properties (continued)

Given two circles of radius a and b, with c as the distance between their centers. If each of the numbers a, b, and c is less than the sum of the other two, then the circles intersect in two points, on opposite sides of the line through the centers.

## SUGGESTED LEARNING ACTIVITIES

1. Overhead transparency - Moise & Downs #27 Circles.

2. Teacher could mimeograph a matching type question using a diagram of a large circle with related parts indicated by a, b, c, etc., to be matched with a given vocabulary list.

3. Filmstrips by FOM #1151

III. CIRCLES

B. SPECIAL ANGLE FORMULAS

## PERFORMANCE OBJECTIVES

The student will:

1. Define selected items from vocabulary list and/or match items from vocabulary list with their given definitions.

2. Identify the kinds of angles related to a circle named in the vocabulary list, given a sketch.

3. Sketch the angles related to a circle, given their names.

4. Develop the angle-measurement formulas listed under "Associated Properties".

5. Apply the angle-measurement formulas to find measures of angles and/or arc given appropriate information with or without a diagram.

6. Solve numerical exercises involving lengths of segments, given appropriate information with or without a diagram.

7. Develop and apply selected associated properties involving chords, radii, secants, tangents, inscribed and circumscribed circles and polygons.

## COURSE CONTENT

Vocabulary

central angle
inscribed angle
angle formed by intersection of two secants intersecting at a point in the interior of the circle
angle with its vertex on a circle, formed by a secant ray and a tangent ray
angle formed by two secants or a secant and a tangent or two tangents of a circle intersecting at a point in the exterior of the circle

degree
degree measure
seconds
minutes
angle inscribed in an arc

III. CIRCLES

B. SPECIAL ANGLE FORMULAS

Associated Properties

If B is a point of $\overset{\frown}{AC}$, then $m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$.

The bisector of a central angle of a circle bisects the corresponding minor arc.

The measure of an inscribed angle is half the measure of its intercepted arc.

Any angle inscribed in a semicircle is a right angle.

All angles inscribed in the same arc are congruent.

If two circles are tangent internally such that the smaller circle contains the center of the larger circle, then any chord of the larger circle having one end point at the point of tangency is bisected by the smaller circle.

If any circle, parallel chords intercept arcs having equal measures.

In a circle, a diameter perpendicular to a chord bisects each arc determined by the end points of the chord.

If an angle inscribed in a circular arc is a right angle, the arc is a semicircle.

The opposite angles of an inscribed quadrilateral are supplementary.

In the same circle or in congruent circles, if two chords are congruent, then so are the corresponding minor arcs.

In the same circle or in congruent circles, if two arcs are congruent, then so are the corresponding chords.

Given an angle with its vertex on a circle, formed by a secant ray and a tangent ray. The measure of the angle is half the measure of the intercepted arc.

If two tangents to a circle intersect each other, they form an isosceles triangle with the chord joining the points of tangency.

III. CIRCLES

B. SPECIAL ANGLE FORMULAS

Associated Properties (continued)

If two arcs are congruent, then any angle inscribed in one of the arcs is congruent to any angle inscribed in the other arc.

The mid-point of the arc intercepted by an angle formed by a secant ray and a tangent ray with its vertex on a circle, is equidistant from the sides of the angle.

The measure of an angle formed by two secants of a circle intersecting at a point in the interior of the circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

The measure of an angle formed by two secants, or a secant and a tangent, or two tangents of a circle intersecting at a point in the exterior of the circle is one-half the difference of the intercepted arcs.

The two tangent segments to a circle from a point of the exterior are congruent and determine congruent angles with the segment from the exterior point to the center of the circle.

## SUGGESTED LEARNING ACTIVITIES

1. Teacher should develop angle formulas in relationship to the position of the vertex.
   - center of circle
   - circle's interior
   - on the circle
   - circle's exterior

2. To review or test, use a ditto question of the type at the bottom of pg. 386 of Modern School Mathematics Geometry by Jurgensen, Donnelly, and Dolciani.

3. Overhead transparency Moise-Downs #28

III. CIRCLES

C. PROPORTIONS INVOLVING CHORDS, SECANTS, AND TANGENTS
(POWER OF A POINT WITH RESPECT TO A CIRCLE)

## PERFORMANCE OBJECTIVES

The student will:

1. Develop properties intuitively and apply them to both numerical and non-numerical exercises.

2. Determine the lengths of tangent segments, secant segments, and segments of chords, by substituting in the appropriate "power of a point" formulas.

3. Determine the measures of angles in a given figu, by applying selected associated properties.

## COURSE CONTENT

### Vocabulary

| | | |
|---|---|---|
| ratio | secant segment | common internal |
| proport.on | tangent segment | tangent |
| "power of a point" | common external tangent | |

### Associated Properties

Given a circle C, and a point Q of its exterior. Let $L_1$ be a secant line through Q, intersecting C in points R and S; and let $L_2$ be another secant line through Q, intersecting C in points U and T. Then

$$QR \times QS = QU \times QT$$

Given a tangent segment $\overline{QT}$ to a circle, and a secant line through Q, intersecting the circle in points R and S. Then

$$QB \times QS = QT^2$$

Let $\overline{RS}$ and $\overline{TU}$ be chords of the same circle, intersecting at Q. Then $QR \times QS = QU \times QT$

III. CIRCLES

C. PROPORTIONS INVOLVING CHORDS, SECANTS, AND TANGENTS
(POWER OF A POINT WITH RESPECT TO A CIRCLE)

Associated Properties (continued)

If the measure of the angle determined by two tangent segments to a circle from a point of the exterior is 60, then the tangent segments form an equilateral triangle with the chord joining the points of tangency.

The common external tangent segments of two circles are congruent.

If two circles and a line intersect in the same point, or points, then the line bisects each common external tangent segment of the circles.

The common internal tangent segments of two points nonintersecting circles are congruent.

## SUGGESTED LEARNING ACTIVITIES

1. In developing the common tangent relationship in circles, use ex. 23 page 459 of Moise-Downs Geometry.

2. Develop the Power of a Point Theorem through proportional sides of similar triangles. Then use "product of the means equals product of the extremes." Convey the idea that the word "power" is a unique number representing the answer to a multiplication probl- .

III. CIRCLES
D. CONSTRUCTIONS

## PERFORMANCE OBJECTIVES

The student will, using only compass and straightedge, construct:

1. A circle containing three given noncollinear points.

2. A circle circumscribed about a triangle.

3. A circle inscribed in a triangle.

4. A center of a given arc.

5. A tangent to a circle from a given external point.

6. A tangent to a circle at a given point on the circle.

7. Three congruent circles, each tangent to the other two, given the radius.

8. Two circles internally tangent, given the raidus of each circle.

9. An equilateral triangle, given the radius of the inscribed circle.

The student will, using only compass and straightedge, try to construct a square which has the same area as a given circle. (Impossible construction -- see Moise, p. 506.)

## COURSE CONTENT

### Vocabulary

external point
noncollinear
isosceles
equilateral

### Associated Properties

Concurrence theorems (angle bisectors, perpendicular bisectors of sides, medians, altitudes of a triangle)

III. CIRCLES
D. CONSTRUCTIONS

Associated Properties (continued)

The Perpendicular Bisector Theorem:
Two points, each equidistant from the endpoints of a given segment, determine the perpendicular bisector of the segment.

A line perpendicular to a radius at its outer end is tangent to the circle.

Every tangent to a circle is perpendicular to the radius drawn to the point of contact.

The set of all points in a plane equidistant from the sides of an angle is the angle bisector.

## SUGGESTED LEARNING ACTIVITIES

1. Filmstrip by Ecuational Projections Inc. #372, Locus.
2. Filmstrip by FOM #1144, Locus problems
3. Do selected constructions using modern drafting implements.
4. Selected students will report on the Impossible Constructions of Antiquity.
5. Throughout this quin, the teacher should include more challenging construction problems from the many sources available in both the State adopted texts and the annotated bibliography at the end of this quin.

III. CIRCLES
E. CIRCUMFERENCE AND AREA

## PERFORMANCE OBJECTIVES

The student will:

1. Find the circumference and/or area of a circle, given the radius or diameter or other necessary information.

2. Find the radius or diameter of a circle, given the circumference and/or area or other necessary information.

3. Find the ratio of the circumferences, areas, or radii of two circles given the measures of their circumferences, areas or radii.

4. Apply the associated properties from III A, B, and C to solve circumference and area problems, including those involving compound figures.

5. Use metric measure in above performance objectives.

## COURSE CONTENT

Vocabulary

annulus apothem

Associated Properties

The circumference of a circle equals pi times the diameter, or two times pi times the radius. ($\pi d$ or $2\pi R$)

The area of a circle equals pi times the square of the radius. ($\pi r^2$)

## SUGGESTED LEARNING ACTIVITIES

1. Compare the areas available when serving dinner from a square, rectangular, and circular table, given a fixed perimeter.

2. Given a constant area, which shaped swimming pool would require the longest concrete walkway? (circular, rectangular, or square)

3. Include an annulus problem (flower bed, etc.)

III. CIRCLES

F. SPECIAL MEASUREMENT FORMULAS

## PERFORMANCE OBJECTIVES

The student will:

1. Find the lengths of arcs, given necessary, information.

2. Find the areas of segments of a circle and sectors, given necessary information.

3. Find missing measures, given lengths of arcs and other necessary information.

4. Find missing measures, given area of segment of a circle or sector of a circle and other necessary information.

## COURSE CONTENT

### Vocabulary

length of ard
proportional
segment of a circle

### Associated Properties

If two arcs have equal radii, then their lengths are proportional to their measures.

If an arc has measure q and radius r, then its length, L, is:

$$L = \frac{q}{180} \cdot r$$

The area of a sector is half the product of its radius and the length of its arc.

If a sector has radius r and its arc has measure q, then it's area is:

$$A = \frac{q}{360} \quad r^2$$

III. CIRCLES

F. SPECIAL MEASUREMENT ACTIVITIES

SUGGESTED LEARNING ACTIVITIES

1. Review algebra skills needed to solve formulas for any one of the variables.

2. Refine students' ability to select the appropriate formula by using a matching type question with a list of formulas to be matched with the proper question.

3. Have students create original designs with and without a spirograph.

4. Have students report on architectural applications of the circle, such as archways, etc.

IV. POLYGONS

A. KINDS AND RELATED PARTS

## PERFORMANCE OBJECTIVES

The student will:

1. Identify convex polygons from assorted convex and non-convex diagrams.

2. Sketch a convex and non-convex polygon of a given number of sides.

3. Name the polygons having 3, 4, 5, 6, 7, 8, 9, 10, 12, and n sides.

4. Determine the number of diagonals of any given convex polygon.

5. Find the sum of the measures of the interior angles of any convex polygon having a given number of sides.

6. Given the sum of the measures of the interior angles, find the number of sides of any convex polygon.

7. Calculate the measures of the interior angles, central angle, and exterior angles of any regular polygon having a given number of sides.

8. Given the measure of an interior angle, central angle, or exterior angle of a regular polygon, find the number of sides.

9. Develop and apply the formula for the sum of the measures of the exterior angles of a convex polygon.

10. Distinguish between regular and non-regular polygons, given appropriate information in a diagram or verbal description.

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTERS | 11, 12, 16 | 10, 17 | 8, 15 | 13, 18 |

IV. POLYGONS

A. KINDS AND RELATED PARTS

## COURSE CONTENT

### Vocabulary

Polygon
- triangle
- quadrilateral
- pentagon
- hexagon
- octagon
- decagon
- duodecagon
- heptagon
- n-gon
- nonagon
- vertex, vertices
- adjacent sides
- consecutive or successive sides and angles

polygonal region
interior
convex polygon
non-convex polygon
diagonal
interior angle
exterior angle
regular polygon
- central angle
- center
- radius
- apothem

### Associated Properties

In any polygon the number of diagonals which can be drawn from any one vertex is (n-3).

In any given polygon the number of triangular regions formed by drawing all diagonals from one vertex is (n-2).

The number of diagonals of any n-gon is $\frac{n}{2}(n-3)$.

The sum of the measures of the interior angles of a convex polygon of n sides is $(n-2) \cdot 180$.

The sum of the measures of the exterior angles of a convex polygon of n sides is 360.

The sum of the measures of the central angles of a convex polygon is 360.

The measure of an interior angle of a regular n-gon is $180 - \frac{360}{n}$ or $\frac{(n-2)\,180}{n}$.

IV. POLYGONS

A. KINDS AND RELATED PARTS

Associated Properties (continued)

The measure of a central angle of a regular n-gon is $\frac{360}{n}$.

The measure of an exterior angle of a regular n-gon is $\frac{360}{n}$.

The number of sides of a regular polygon is $\frac{360}{\text{measure of exterior angle}}$ or $\frac{360}{\text{measure of central angle}}$.

The only three regular polygons which may be repeated to fill out a plane are an equilateral triangle, a square, and a regular hexagon.

In any given polygon, the number of sides, interior angles, exterior angles and central angles (in reg. polygon only) is the same.

## SUGGESTED LEARNING ACTIVITIES

1. Filmstrip by FOM #1140 Angle sums for Polygons

2. Overhead transparency, Moise-Downs #29

3. Selected students can report on "space filling figures." Refer to pages 19-21 of Irving Adler's "A New Look at Geometry".

4. Have students create original designs using regular polygons.

IV. POLYGONS

B. CONGRUENT AND SIMILAR POLYGONS

## PERFORMANCE OBJECTIVES

The student will:

1. State and apply the definitions of similar polygons and congruent polygons.

2. Enlarge or reduce the size of a given polygonal drawing according to a given scale.

3. Construct a polygon congruent to a given polygon.

4. Given similar polygons and the measures of some of their sides, find the lengths of the other corresponding sides.

5. Determine if polygons are similar under a given set of conditions.

## COURSE CONTENT

### Vocabulary

| | |
|---|---|
| irrational | reflexive |
| scale drawing | symmetric |
| similar polygons | transitive |
| congruent polygons | equivalence |

### Associated Properties

Similarity of convex polygons is reflexive, symmetric, and transitive, and is, therefore, an equivalence relation. (See Moise, Pg. 118)

All congruent figures are similar, but not all similar figures are congruent.

## SUGGESTED LEARNING ACTIVITIES

1. Cut out carboard models to match and compare shapes and sizes.

2. Students should research Home magazine for "odd" shaped floor plans.

IV. POLYGONS

B. CONGRUENT AND SIMILAR POLYGONS

3. Have students develop original plans for a room, house, table, etc.

4. Students can develop original tile designs for a floor and research Turkish & Indian tile designs using familar polygons.

IV. POLYGONS

C. PERIMETER AND AREA

## PERFORMANCE OBJECTIVES

The student will:

1. Determine the perimeter of a polygon given the length of its sides.

2. Determine the perimeter of a regular polygon given the length of one side.

3. Determine the area of a regular polygon given sufficient information.

4. Determine the perimeter and area of compound figures, given the necessary measures.

5. Given the area of a regular polygon and other necessary measures of the polygon, find the missing measures of other parts.

6. Determine the area of a regular octagon, given the radius of the octagon.

7. Determine the area of a regular hexagon, given the radius of the hexagon.

8. Develop and apply the special formula for the area of a regular hexagon $\frac{ap}{2}$.

9. Determine the radius, circumference, and area of the inscribed and circumscribed circles, given appropriate measures of a regular polygon.

10. Determine the ratio of their perimeters and areas given the ratio of the apothems or sides of two similar polygons, and vice versa.

11. Determine the area of a regular hexagon given the perimeter, and vice versa.

12. Develop and apply the formula for the area of a regular polygon.

13. Apply metric measure, where appropriate, to above objectives.

IV. POLYGONS

C. PERIMETER AND AREA

## COURSE CONTENT

### Vocabulary

regular
limit
compound figures

### Associated Properties

The area of a regular hexagon is $\frac{3\sqrt{3}\,s^2}{2}$

The ratio of the areas of similar polygons equals the square of the ratio of their corresponding apothems, sides, radii, perimeters.

The perimeter of a regular polygon equals the product of the length of a side times the number of sides.

The area of a regular polygon equals 1/2 the product of the length of the apothem and perimeter.

As the number of sides of a regular polygon inscribed in a circle increases, its perimeter approaches the circumference of the circumscribed circle; its apothem approaches the radius of the circumscribed circle; the length of its sides approaches zero; the measure of an interior angle approaches 180; the area of the inscribed polygon approaches the area of the circumscribed circle.

## SUGGE TED LEARNING AC TIVITIES

1. Use selected designs & floor plans from IV B, assign appropriate measures, then have students compute perimeters & areas.

2. Refer to Tanagrams & games such as "Pythagoras" and "Euclid".

3. Students can create their own tanagrams starting with a piece of graph paper and a square 20 boxes by 20 boxes. They are to cut 5 or 6 different shapes from that square, compute each area, and add them up again to match the area of the original square.

4. Use geoboard to create and compute measurements of various polygonal shapes.

IV. POLYGONS

D. CONSTRUCTIONS

## PERFORMANCE OBJECTIVES

Using only a compass and straightedge, the student will construct:

1. a circle inscribed in a regular polygon.
2. a circle circumscribed about a regular polygon.
3. a regular hexagon.
4. an equilateral triangle inscribed in a circle.
5. a square inscribed in a circle.
6. a regular octagon inscribed in a circle.
7. a polygon congruent to a given polygon.
8. a smaller or larger polygon similar to a given polygon using a given ratio of the lengths of the sides.

## COURSE CONTENT

### Vocabulary

inscribed polygon
circumscribed polygon

### Associated Properties

A circle can be inscribed in and circumscribed about, any regular polygon.

In a circle, the length of a side of an inscribed regular hexagon equals twice the apothem of an inscribed equilateral triangle.

The radius of a regular hexagon is equal to the length of its side.

IV. POLYGONS

D. SPECIAL CONSTRUCTIONS

SUGGESTED LEARNING ACTIVITIES

1. Any mechanical drawing or graphics book will provide excellent additional problems for this unit.

2. Use "Kaleidoscope Geometry" to construct various polygons, as explained in Readings in Geometry an NCTM publications mentioned in the annotated bibliography.

3. Throughout this quin, the teacher should inlcude more challenging construction problems from the State-adopted texts, and the sources listed in the annotated bibliography.

V. THREE-DIMENSION FIGURES (SOLIDS)
A. KINDS AND RELATED PARTS

## PERFORMANCE OBJECTIVES

The student will:

1. Describe the method of generation and the required parts for a given solid.

2. Sketch the solids named in the vocabulary list and identify their parts.

3. Match items in a verbal list of solids and related parts with items given pictorially.

## STATE ADOPTED REFERENCES

| | M | L | JD | A |
|---|---|---|---|---|
| CHAPTERS | 17 | 18 | 16 | 20 |

## ADDITIONAL REFERENCES

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## V. THREE-DIMENSIONAL FIGURES (SOLIDS)
### A. KINDS AND RELATED PARTS

Vocabulary

| | | |
|---|---|---|
| solid | edge | right circular cylinder |
| prism | cross section | right circular cone |
| pyramid | plane section | cylinder of revolution |
| cylinder | polyhedral angle | tangent plane |
| cone | triangular prism | parallelepiped |
| right prism | tetrahedron | rectangular |
| lateral surface | regular solid | parallelepiped |
| total surface | Platonic solid | cube |
| interior | frustum | similar solids |
| exterior | slant height | congruent solids |
| regular pyramid | lateral face | dodecahedron |
| axis | lateral edge | icosahedron |
| base | regular octahedron | plantonic solids |
| face | circular cylinder | volume |

Associated Properties

All cross sections of a prism are congruent to the base.

The lateral faces of a prism are parallelogram regions.

The lateral faces of a right prism are rectangular regions.

Dual Solids: The centers of the six faces of a cube are the verticies of an octahedron. The centers of the eight faces of an octahedron are the vertices of a cube. The dodecahedron and the icosahedron are also dual solids.

The five regular or platonic solids are:

cube - six faces (squares)
eight vertices
twelve edges

tetrahedron - four faces (equilateral triangles)
four vertices
six edges

octahedron - eight faces (equilateral triangles)
six vertices
twelve edges

V. THREE-DIMENSIONAL FIGURES (SOLIDS)
A. KINDS AND RELATED PARTS

Associated Properties (continued)

| | |
|---|---|
| dodecahedron - | twelve faces (regular pentagons)<br>twenty vertices<br>thirty edges |
| icosahedron - | twenty faces (equilateral triangles)<br>twelve vertices<br>thirty edges |

SUGGESTED LEARNING ACTIVITIES

1. Students can make models of the various polyhedrons as discussed in a book by Wenninger, Magnus J. Polyhedron Models for the Classroom. Washington, D.C. 1970.

V. THREE-DIMENSIONAL FIGURES (SOLIDS)
B. AREA (LATERAL, TOTAL, CROSS-SECTIONAL)

## PERFORMANCE OBJECTIVES

The student will:

1. Compute the lateral, total or cross-section area of any prism, pyramid, cone, cylinder, or sphere given the appropriate measures.

2. Compute the lateral and total area of a compound figure given the necessary measures.

3. Compute the length of an edge, the slant height, the altitude, etc., given the lateral, total, or cross-section area of a certain solid and other necessary measures.

4. Compute the area of a cross-section of a solid a given distance from the base of the solid, given the altitude and base of the solid.

5. Compare cross-sectional areas of the same solid given the necessary measurements.

6. Compare lateral edges, altitudes and bases of two similar solids, given the ratio of the areas of two corresponding cross-sections.

7. Apply metric measure where appropriate to the above objectives.

## COURSE CONTENT

### Vocabulary

area
lateral area (LA)
total area (TA)
cross sectional area
slant height (l)
radius (r)
length of base edge (e)
altitude (h)
area of Base (B)
perimeter of Base(p)

### Associated Properties

Right Prism
$LA = ph$
$TA = ph + 2B$

Associated Properties (continued)

Regular Pyramid
$LA = 1/2\ pl$
$TA = 1/2\ pl + B$

Right circular cylinder
$LA = 2\pi rh$
$TA = 2\pi (rh + r^2)$

Right Circular Cone
$LA = \pi rl$
$TA = \pi rl + \pi r^2$

Sphere
$SA = 4\pi r^2$

Frustum of a regular pyramid
$LA = (1/2)\ l\ (p_1 + p_2)$

Frustum of a right circular cone
$LA = (1/2) l\ (2\pi r_1 + 2\pi r_2) = \pi l\ (R_1 + R_2)$

The lateral areas and total surface areas of 2 similar solids have the same ratio as the squares of the lengths of any pair of corresponding altitudes, slant heights, radii, edges, etc.

Every cross section of a triangular pyramid, between the base and the vertex, is a triangular region similar to the base. If h is the altitude, and k is the distance from the vertex to the cross section, then the area of the cross section is equal to $k^2/h^2$ times the area of the base.

In any pyramid, the ratio of the area of a cross section to the area of the base is $k^2/h^2$, where h is the altitude of the p pyramid and k is the distance from the vertex to the plane of the cross section.

If two pyramids have the same base area and the same altitude, then cross sections equidistant from the vertices have the same area.

Every cross section of a circular cylinder has the same area as the base.

Given a cone of altitude h, and a cross section made by a plane at a distance k from the vertex. The area of the cross section is equal to $k^2/h^2$ times the area of the base.

V. THREE-DIMENSIONAL FIGURES (SOLID)
C. VOLUMES

## PERFORMANCE OBJECTIVES

The student will:

1. Compute the volume of a certain solid given the appropriate measurements.

2. Compute the length of a selected segment of area, given the volumes of a certain solid and other needed measurements.

3. Find the ratio of the volumes of similar solids given the ratio of their corresponding segments and vice versa.

4. Develop intuitively and apply Cavalieri's Principle.

5. Find the volume of the second of two solids, given the volume of the first solid and an appropriate ratio of a pair of corresponding parts of the two solids.

6. Apply the metric measure, where appropriate, to above objectives.

## COURSE CONTENT

### Vocabulary

volume (v)
altitude (h)
area of base (B)
edge (e)
radius (r)

### Associated Properties and Formulas

Frustum $= \frac{1}{3} h (B_1 + B_2 + \sqrt{B_1 B_2})$

Right Prism $V = (Bh)$

Regular Pyramid $V = \frac{1}{3} Bh$

Right Circular Cylinder $V = \pi r^2 h$

V. THREE-DIMENSIONAL FIGURES (SOLID)
C. VOLUMES

<u>Associated Properties and Formuals</u> (continued)

Right Circular Cone $V = \frac{1}{3}\pi r^2 h$

Sphere $V = \frac{4}{3}\pi r^3$

The volumes of similar solids have the same ratio as the cubes of the lengths of any pair of corresponding altitudes, slant heights, edges, radii, etc.

Cavalieri's Principle: Given two solids and a p ane, ppose that every plane parallel to the given plane, inte ing one of the two solids, also intersects the other, and gives cross sections with the same area, then the two solids have the same volume.

## <u>SUGGESTED LEARNING ACTIVITIES</u>

Dade County Experimental Projects for Geometry Level I

Unit 3: Identifying Shapes

Topic: Polyhedron Models

Topic: Shadows

## SAMPLE POSTTEST ITEMS

### I. TRIANGLES

TRUE-FALSE. Correct the underlined portion of each of the false statements.

1. The sum of the measures of the interior angles of a triangle is half the sum of the measures of its three exterior angles formed by extending the sides of a triangle in one direction.

2. In any triangle, the three medians intersect in a common point called the <u>centroid.</u>

3. The segment joining the midpoints of two sides of a triangle is <u>twice</u> the length of the third side.

4. Two triangles can be proved congruent if three angles of one are respectively congruent to three angles of the other.

5. The bisector of any angle of a <u>scalene</u> triangle divides the triangle into two congruent triangles.

6. The shortest side of a 30-60-90 triangle is <u>one-third</u> the length of the hypotenuse.

7. Two angles of a triangle are 40 and 60. The number of degrees in the obtuse angle formed by the bisectors of these two angles is <u>130.</u>

8. Two angles that are both congruent and supplementary must be <u>right</u> angles.

9. The bisectors of the angles of a triangle meet at a point which is equidistant from the three <u>vertices</u> of the triangle.

10. Two of the exterior angles of a right triangle must be <u>acute.</u>

### II. QUADRILATERALS

1. An equilateral quadrilateral that is not equiangular is called a(n) ____________________.

2. If the diagonals of a parallelogram are not congruent and are perpendicular to each other, the figure must be a ___________.

## SAMPLE POSTTEST ITEMS
(continued)

II. QUADRILATERALS (continued)

3. If the diagonals of a quadrilateral are congruent and bisect each other, the figure must be a ____________.

4. If the ratio of the measures of two angles of an isosceles trapezoid is 4:1, the measures of the angles are ______ and ______.

5. If the bases of a trapezoid measure 12 inches and 18 inches, find the length of the segment joining the midpoints of the non-parallel sides.

6. Name the figure formed by the segments joining the midpoints of the consecutive sides of any quadrilateral.

TRUE - FALSE. Correct the underlined portion of each of the false statements.

7. If the diagonals of a rectangle are perpendicular to each other, the figure must be a <u>square.</u>

8. If the diagonals of a parallelogram are congruent, the figure must be a <u>square</u>.

9. A parallelogram must be a <u>rhombus</u> if it is equilateral.

10. If the midpoints of two adjacent sides of a rhombus are joined, the triangle formed must be <u>equilateral.</u>

III. CIRCLES

For each of the following statements indicate whether the information given in the hypothesis is (a) too little (b) just enough or (c) too much to justify the conclusion.

1. If from one end of a diameter two congruent chords are drawn, they will be equidistant from the center.

2. The measure of the angle formed by the intersection of two tangents is equal to one-half the difference of the measures of the intercepted arcs.

3. Two equilateral triangles inscribed in a circle are congruent.

III. CIRCLES (continued)

4. If two chords of a circle are congruent and parallel, then the chord passing through their midpoints is a diameter.

5. In the same or congruent circles, congruent chords are equidistant from the center of the circle(s).

6. The sum of the measures of the opposite arcs intercepted by two chords which meet inside a circle to form vertical angles of 50 degrees is 100 degrees.

7. Tangent segments $\overline{PA}$ and $\overline{PB}$ are drawn from an external point P to points A and B of circle 0. They form an angle of 70 degrees. If radii $\overline{OA}$ and $\overline{OB}$ are drawn, the measure of $\angle$ AOB is 110 degrees.

8. If a secant and a tangent to a circle are parallel, the diameter drawn to the point of tangency is perpendicular to the secant.

9. If two chords of a circle are perpendicular to each other, one is a diameter.

10. If two tangents are drawn to a circle from an external point and the major arc intercepted by the tangents is twice the minor arc, then the measure of the angle formed by the tangents is 60.

IV. POLYGONS

1. Using a given segment as a side, construct a regular hexagon.

2. Using a given segment as the radius of the circumscribed circle, construct a regular octagon.

Complete each of the following statements:

3. Two regular pentagons are always ________.

4. If a polygon has ten sides, then it is called a ____________.

5. Regular polygons of the same number of sides are ____________.

## SAMPLE POSTTEST ITEMS
(continued)

IV. POLYGONS (continued)

TRUE - FALSE

6. If a regular hexagon is inscribed in a circle whose radius is 10 inches, the difference between the area of the circle and the area of the hexagon (Correct to the nearest square inch) is 55 square inches. (pi = 3.14 and $\sqrt{3} = 1.73$)

7. The number pi is a constant which represents the ratio between the circumference and the diameter of the circle.

8. The regular polygon whose apothem is one-half its side is a square.

9. The number of degrees in each interior angle of a regular hexagon is 120.

10. The number of degrees in each exterior angle of a regular octagon is 135.

V. SOLIDS

1. A cylindrical gasoline tank is 3.5 feet long and 16 inches in diameter. What is the volume of the tank?

2. Find the volume of a water tank 20 inches deep if its bottom is in the form of a rectangle with a semicircle at each end, and the rectangle is 18 inches wide and 80 inches long.

3. Find the length of a bar that can be made from a cubic foot of iron, if the bar has a rectangular cross section 1 inch by 1 - 1/4 inches.

4. Find the volume of a cube whose area is 600 square inches.

5. Two corresponding edges of two similar tetrahedrons measure 3 inches and 5 inches respectively. Compare their volumes.

6. The volume of one polyhedron is 216 cubic inches and its total area is 108 square inches. If the total area of a similar polyhefron is 294 square inches, what is its volume?

V. SOLIDS (continued)

7. If a polyhedron has 8 faces and two more than twice as many edges, how many vertices does it have?

8. Find the volume of a regular octahedron if its total area is $72\sqrt{3}$ square inches.

9. The bases of the frustum of a pyramid are squares having sides of four inches and nine inches respectively. Find the volume of the frustum if its altitude is 7 7/9 inches.
$V = \frac{1}{3} h (B_1 + B_2 + \sqrt{B_1 B_2})$

10. Show that the volume of a solid formed by revolving an equilateral triangle with a side "s" about an altitude is $\frac{1}{24} \pi s^3 \cdot \sqrt{3}$.

## KEY TO SAMPLE POSTTEST ITEMS

### I. TRIANGLES

1. True
2. True
3. False - Half
4. False - Similar
5. False - Equilateral
6. False - One-half
7. True
8. True
9. False - Sides
10. False - Obtuse

### II. QUADRILATERALS

1. Rhombus
2. Rhombus
3. Rectangle
4. $36^{o}$ and $144^{o}$
5. 15 inches
6. Parallelogram
7. True
8. False - Rectangle
9. True
10. False - Isosceles

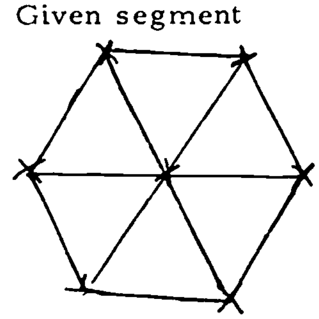
### III. CIRCLES

1. c
2. a
3. b
4. b
5. b
6. b
7. True
8. True
9. False
10. True

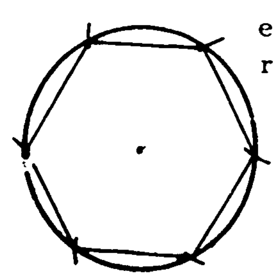
### IV. POLYGONS

1. Construct an equilateral △ using given segment as length of a side. Continue the construction of equilateral △ until six have been completed.

Given segment

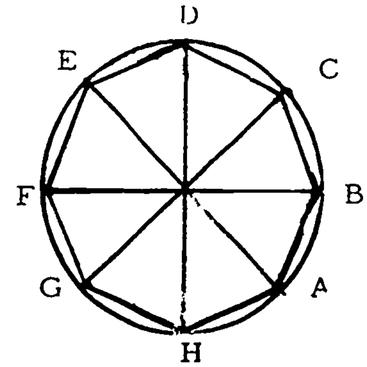


or Construct a circle having given segment as radius. Use compass to mark off 6 chords around circle equal in length to the radius of the circle.



IV. (continued)

2. Given segment




Construct circle having radius equal to length of given segment. Draw diameter. Construct $\perp$ bisector of diameter. Bisect 2 adjacent right angles and extend the bisectors to intersect circle in two points. Connect points of intersection on circle. (A, B, C, D, E, F, G, H)

3. Similar
4. Decagon
5. Similar
6. True
7. True
8. True
9. True
10. False

V. SOLIDS

1. $\frac{56\pi}{9}$ cubic feet

2. $(28,800 + 1620\pi)$ cubic inches

3. $115\frac{1}{5}$ feet

4. 1,000 cubic inches

5. 27 : 125

6. $686\sqrt{2}$ cubic inches

7. 12

8. $72\sqrt{2}$ cubic inches

9. $267\frac{1}{27}$ cubic inches $(V = \frac{1}{3} h \ (B + b + \sqrt{Bb})$

10. Solid will be a cone. Area of base = $\pi\left(\frac{s}{2}\right)^2$

$$V = \frac{1}{3} \cdot \frac{\pi s^2}{4} \cdot \frac{s\sqrt{3}}{2} = \frac{1}{24}\pi s^3 \sqrt{3}$$

## ANNOTATED BIBLIOGRAPHY
(Not on State Adopted List)


Adler, Irving. A New Look at Geometry. New York: The New American Library, Inc., 1967. (A Signet Book).

Adler, Irving. Mathematics - The Story of Numbers, Symbols and Space. New York: Golden Press, 1961.
A very basic, elementary but interesting treatment of many of the items covered in this quin. This is an excellent source book for students.

Dade County Experimental Project for Geometry Level I.
Excellent for activities and procedures developing intuitive conclusions.

Dodes, Irving A. Geometry. New York: Harcourt, Brace and World, Inc., 1965.
Good for practical and commercial applications of geometry. Especially chapt. 4, 6-9, 11-14.

Fehr, Howard F. and Carnahan, Walter H. Geometry. Boston Mass: D.C. Heath and Co., 1961.
Chpts. 3, 6, 7, 9-15, 17-19 excellent for test questions and quiz material. Very good intuitive approach in the Discovery Exercises.

Gardner, Martin. Mathematical Puzzles and Diversions. New York: Simon and Schuster, 1959.
Marvelous source for the curious and resourceful student looking for something different.

Goodwin, Wilson, A. and Vannatta, Glen D. Geometry. Columbus, Ohio: Charles E. Merrill Publishing Co., 1970.
Chpts. 6, 8-13, 16-19 all very helpful in this quin. Excellent 3-D drawings and explanation.

Keedy, Marvin, L. and Jameson, Richard, E Exploring Geometry. New York: Holt, Rinehart, and Winston, Inc., 1967.
Excellent diagrams and explanations of three-dimensional figures. Teachers commentary gives helpful additional explanations. Very good source book for entire quin. Excellent "applied" problems in each chapter.

Menger, Karl. You Will Like Geometry. Chicago, Ill: Illinois Institute of Technology, 1961.
Another interesting and readable book for the students as well as the teacher.

Munro, Thomas and Wilson, Catherine M. Tenth Year Mathematics. New York: Oxford Book Co., 1960.
Good basic review material. Excellent for additional questions for homework or tests. Explanations very brief and basic.

N.C.T.M. Geometry, Unit Four of Experiences in Mathematical Discovery (Washington D.C.: National Council of Teachers of Mathematics, 1966.
Concise but excellent references on many points covered in this quin from the intuitive and discovery angle.

N.C.T.M. Readings in Geometry from the Arithmetic Teacher. Washington, D.C.: National Council of Teachers of Mathematics, 1971.
Excellent understandable explanation of fun topics covered in this quin. For both teacher and student, a delightful introduction to unusual elementary approaches.

Ransom, William. Can and Can't in Geometry. Portland, Maine: J. Weston Walch, 1960.
Unusual explanations of quin topics useful for teacher research.

Smith, Rolland R. and Ulrich, James F. Geometry, A Modern Course. New York: Harcourt, Brace and World, Inc., 1964.
Excellent chapter on constructions and three-dimensional geometry. (Chapter 3). Chpts. 10, 11, 15, 17, 19, 20 all very helpful in this quin.

Ulrigh, James, F., and Payne, Joseph N. Geometry. New York: Harcourt, Brace and World, Inc., 1969. Chpts. 5, 7, 8, 10, 12, 14
Excellent for definitions and explanations of work covered in this quin. Chapter 11 covers the Pythagorean Theorem in depth.

Wilcox, Marie S. Geometry a Modern Approach. Reading, Mass: Addison-Wesley Publishing Co., 1968.
Good for quiz and test materials as well as general spacial concepts. Chpts. 5, 8, 11-14 apply particularly to this quin.

Annotated Bibliography (continued)

## AUDIO-VISUAL MATERIAL

Films - By request only through the school library

Cat. Number

1-01506 Similar Triangles (12 min.)

1-01348 Triangles: Types and Uses (11 min.)

Curriculum Full-Color Filmstrips by Educational Projections, Inc.

366 Triangles. Types of triangles, median, angle bisection, altitude, hypotenuse, exterior angles.

372 Locus. Very good explanation and examples.

Filmstrips by S.V.E.

A542-4 Congruence

Filmstrips by McGraw-Hill Book Co.

9 Indirect measurement. (Excellent) Use after study of right triangles.

Set 1; 7 & 8 Constructions with compass

Set 1; Geometric Figures

Filmstrips by PSP (Pop. Sci. Pub. Co. NYC)

1181 Right Angles in 3-D Figures

1155 Applied Geometry

Filmstrips by FOM (Pop. Sci. Pub. Co., NYC)

1168 Special right triangles

1113 Parallelograms and their properties

1122 Congruent triangles

1140 Angle sums for polygons

Filmstrips by FOM (continued)

1161 Mean proportion and right triangles

1151 Diameter perpendicular to a chord

1116 Similar triangles - Experiment and deduction

1157 Using congruence in construction

1144 Locus problems: the circle

OVERHEAD VISUALS

Overhead Visuals for Geometry by Jurgensen, Donnelly, Dolciani

Vol. III Parallel Lines and Planes
Congruent Triangles

Vol. V Construction and Loci

OVERHEAD TRANSPARENCY MASTERS & TEACHERS GUIDE TO ACCOMPANY GEOMETRY MOISE-DOWNS IN GREEN BOX